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# Diquark condensation at strong coupling

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Abstract: The possibility of diquark condensation at sufficiently large baryon chemical potential and zero temperature is analyzed in QCD at strong coupling. In agreement with other strong coupling analysis, it is found that a first order phase transition separates a low density phase with chiral symmetry spontaneously broken from a high density phase where chiral symmetry is restored. In none of the phases diquark condensation takes place as an equilibrium state, but, for any value of the chemical potential, there is a metastable state characterized by a non-vanishing diquark condensate. The energy difference between this metastable state and the equilibrium state decreases with the chemical potential and is minimum in the high density phase. The results indicate that there is attraction in the quark-quark sector also at strong coupling, and that the attraction is more effective at high baryon density, but for infinite coupling it is not enough to produce diquark condensation. It is argued that the absence of diquark condensation is not a peculiarity of the strong coupling limit, but persists at sufficiently large finite couplings.

Keywords: Spontaneous Symmetry Breaking, Strong Coupling Expansion, Lattice Gauge Field Theories, Lattice QCD.

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## 1. Introduction

Hadron phenomenology suggests that quarks confined within the baryons are strongly clustered into diquarks [1, 2]. Theoretically, a weak coupling analysis of QCD at the level of one gluon exchange shows that the quark-quark scattering amplitude is attractive in the color anti-triplet channel and repulsive in the symmetric color sextet channel. The attraction in the color anti-triplet channel due to one gluon exchange provides a mechanism for bounding the diquark in a state the color of which is neutralized by the remaining quark within the baryon. The quark-quark interaction induced by non-perturbative instanton effects produces also attraction in the color anti-triplet channel and repulsion in the color sextet channel.

The effect of the quark-quark attraction should be more dramatic at high baryon densities [3]. At zero density the loosely bound diquarks are in its turn bound to quarks producing the color singlet baryons. At high baryon density, however, the color charge is expected to be deconfined and the weakly interacting quarks may form a Fermi surface that the quark-quark attraction will render unstable. Then, the phenomenon of color superconductivity would take place via a BCS mechanism, with the formation of a diquark condensate [3]–[5]. The diquark condensate can be studied in QCD in the weak coupling regime via the Schwinger-Dyson equations [6]–[9] and instabilities of vertex functions [10]. The diquark condensation offers the possibility of exotic phenomena at high baryon density, as color-flavor locking and unlocking and crystalline superconductivity [11]–[14], and the continuity of the quark and hadronic phases [15].

The diquark operator carries color charge and is not gauge invariant. Hence, general arguments forbide diquark condensation in the naive way [16] (see also [17]). Diquark condensation must be understood not as the spontaneous breaking of the gauge symmetry, but as a kind of Higgs mechanism that has the observable consequencies of color superconductivity [18, 19].



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Recently, strong evidence of diquark condensation in two colors QCD has been found [20]–[25]. The analysis in this case is simpler than in three colors QCD for two reasons: there is a gauge invariant diquark operator and the fermionic determinant is positive at finite chemical potential, so that Monte Calo simulations are feasible.

In three colors QCD, however, the arguments in favour of diquark condensation at high density are based on weak coupling analysis (although nonperturbative effects are taken into account via instantons), that should be valid at asymptotically large densities. It is very important to verify the diquark condensation relaxing the weak coupling assumptions. Unfortunately, numerical simulations of lattice QCD at finite density are unfeasible ought to the sign problem. Hence, the strong coupling techniques are valuable to get insight in the problem. Recent attempts to study QCD at finite density in the strong coupling limit by using hamiltonian techniques have been developed in [26]–[28]. In this paper, we shall study the possibility of diquark condensation in the strong coupling limit using the path integral formalism.

## 2. Effective action at strong coupling

Let us consider QCD regularized on a euclidean four dimensional lattice with one flavor of staggered fermions. The gauge group is SU(3). Let us define the meson and baryon operators:

$$\begin{aligned} M(x) &= \bar{\psi}^a(x)\psi^a(x) \\ B(x) &= \frac{1}{6}\epsilon_{abc}\psi^a(x)\psi^b(x)\psi^c(x) \\ \bar{B}(x) &= \frac{1}{6}\epsilon_{abc}\bar{\psi}^a(x)\bar{\psi}^b(x)\bar{\psi}^c(x)\,, \end{aligned} \tag{2.1}$$

where summation over repeated color indexes, $a, b, c$, is understood. At strong coupling, $1/g^2 = 0$, the gauge field can be integrated out and gives the following effective action for the fermions [29]:

$$\begin{aligned} S_{\rm sc}^{\rm eff} = m_0 \sum_x M(x) - \frac{1}{2}\sum_{x,y} M(x)V_{\rm M}(x,y)M(y) - \sum_{x,y}\bar{B}(x)V_{\rm B}(x,y)B(y) + \\ + \frac{1}{576}\sum_{x,\nu} M^2(x)M^2(x+\nu) - \frac{5}{576}\sum_{x,\nu}\bar{B}(x)B(x)\bar{B}(x+\nu)B(x+\nu)\,, \end{aligned} \tag{2.2}$$

where $m_0$ is the bare fermion mass and

$$\begin{aligned} V_{\rm M}(x,y) &= \frac{1}{12}\sum_\nu(\delta_{y,x+\nu} + \delta_{y,x-\nu})\,, \\ V_{\rm B}(x,y) &= \frac{1}{8}\sum_\nu[f_\nu(x)\delta_{y,x+\nu} - f_\nu(x)^{-1}\delta_{y,x-\nu}]\,, \end{aligned} \tag{2.3}$$

where $\nu$ are the unit vectors in the four space time directions and

$$f_\nu(x) = \begin{cases} e^{3\mu}, & \nu = 0 \\ \eta_\nu(x), & \nu = 1,2,3 \end{cases}, \tag{2.4}$$

where $\mu$ is the baryon chemical potential and $\eta_\nu(x)$ are the Kogut-Susskind phases.

The zero temperature grand canonical partition function is given by

$$\mathcal{Z} = \int [d\bar{\psi} d\psi] \; \exp[-S_{\rm sc}] \,. \tag{2.5}$$

We will neglect the last two terms of the fermion effective action, which involve eight-fermion and twelve-fermion vertices. It has been proved that these terms are sub-dominant in a $1/d$ expansion, where $d$ is the space-time dimension [29]. In $d = 4$ their contribution to the thermodynamics is small and does not change qualitatively the phase diagram [31]. We linearize the remaining meson and baryon terms in the usual way, introducing a bosonic field, $\varphi(x)$, and two fermion fields, $b_x$ and $\bar{b}_x$, via the Hubbard-Stratonovich transformations:

$$\begin{aligned}
\exp\left\{\frac{1}{2}(M, V_{\rm M} M)\right\} &= (\det V_M)^{-1/2} \int [d\varphi] \exp\left\{-\frac{1}{2}(\varphi, V_{\rm M}^{-1}\varphi) - (\varphi, M)\right\}, \\
\exp\{(\bar{B}, V_{\rm B} B)\} &= \det V_B \int [d\bar{b} db] \exp\left\{-\frac{1}{2}(\bar{b}, V_{\rm B}^{-1} b) \; + \; (\bar{b}, B) + (\bar{B}, b)\right\}.
\end{aligned} \tag{2.6}$$

where we use the notation $(f, Ag) = \sum_{xy} f(x) A(x,y) g(y)$ and $(f,g) = \sum_x f(x) g(x)$.

Let us introduce the diquark fields

$$D^a(x) = \epsilon_{abc} \psi^b(x) \psi^c(x) \tag{2.7}$$

$$\bar{D}^a(x) = \epsilon_{abc} \bar{\psi}^b(x) \bar{\psi}^c(x) \,. \tag{2.8}$$

Note that $\bar{D}^a(x)$ transforms as a quark (color triplet) and $D^a(x)$ as an antiquark (color anti-triplet) under gauge transformations. It is convenient to our purpose to rewrite the term $\exp[(\bar{b}, B) + (\bar{B}, b)]$ entering equation (2.6) by using the following identity:

$$\begin{aligned}
\exp[\bar{b}_x B(x) + \bar{B}(x) b_x] &= \exp\left[\frac{1}{36}\bar{b}_x b_x M(x) + 2M^2(x)\right] \int d\phi^\dagger(x) d\phi(x) \times \\
&\quad \times \exp\Bigg\{-\phi_a^\dagger(x)\phi_a(x) - \phi_a^\dagger(x)\left[\frac{1}{6}\bar{b}_x \psi^a(x) + \bar{D}^a(x)\right] - \\
&\quad - \phi_a(x)\left[\frac{1}{6}\bar{\psi}^a(x) b_x + D^a(x)\right]\Bigg\},
\end{aligned} \tag{2.9}$$

where $\phi_a(x)$ is a complex bosonic field that carries color charge in the fundamental representation. The above series of transformations lead to the following expression for the grand canonical partition function:

$$\mathcal{Z} = \int [d\bar{\psi} d\psi][d\bar{b} db][d\varphi][d\phi^\dagger d\phi] \; \exp[-S_{\rm fb}^{\rm eff}] \,, \tag{2.10}$$

where

$$\begin{aligned}
S_{\rm fb}^{\rm eff} &= \sum_x \phi_a^\dagger(x)\phi_a(x) + \frac{1}{2}\sum_{x,y}\varphi(x) V_{\rm M}^{-1}(x,y)\varphi(y) + \sum_{x,y} \bar{b}_x V_{\rm B}^{-1}(x,y) b_y - \ln\det V_B + \\
&\quad + \sum_x \Bigg\{\left(m + \varphi(x) - \frac{1}{36}\bar{b}_x b_x\right) M(x) - 2M^2(x) + \phi_a^\dagger(x)\left[\frac{1}{6}\bar{b}_x\psi^a(x) + \bar{D}^a(x)\right] + \\
&\quad + \phi_a(x)\left[\frac{1}{6}\bar{\psi}^a(x) b_x + D^a(x)\right]\Bigg\}.
\end{aligned} \tag{2.11}$$

Notice that the scalar field $\phi_a(x)$ acts as a Higgs field with color charge in the fundamental representation.

The integral over the fermion fields $\psi^a(x)$ and $\bar{\psi}^a(x)$ factorizes as a product of integrals at each lattice site and can be readily performed. Afterwards, the auxiliary fields $b_x$ and $\bar{b}_x$ can be integrated out, and we arrive to a representation of the grand canonical partition function as a functional integral over bosonic fields:

$$\mathcal{Z} = \int [d\varphi][d\phi^\dagger d\phi] \, \exp[-S_{\rm b}^{\rm eff}] \,, \tag{2.12}$$

with

$$S_{\rm b}^{\rm eff} = \sum_x \phi_a^\dagger(x)\phi_a(x) + \frac{1}{2}\sum_{x,y} \varphi(x) V_{\rm M}^{-1}(x,y)\varphi(y) - \ln\det[\tilde{\Theta}_1 - V_{\rm B}^{-1}\tilde{\Theta}_2] \,, \tag{2.13}$$

where $\tilde{\Theta}_i(x,y) = \Theta_i(x)\delta_{xy}$, $i = 1,2$, with

$$\Theta_1(x) = m(x)[m^2(x) - 4|\phi(x)|^2 + 12] \,, \tag{2.14}$$

$$\Theta_2(x) = \frac{1}{36}[4|\phi(x)|^4 + m^2(x)|\phi(x)|^2 + 8|\phi(x)|^2 - 3m^2(x) - 12] \,. \tag{2.15}$$

In the above equation $m(x) = m_0 + \varphi(x)$.

The field $\varphi(x)$ is related to the chiral condensate in the usual way, by

$$\langle M(x)\rangle = c\langle\varphi(x)\rangle \,, \tag{2.16}$$

where $c = (1/V)\sum_{xy} V_{\rm M}^{-1}(x,y)$, with $V$ the lattice volume. Hence, a non vanishing vacuum expectation value of $\varphi(x)$ implies spontaneous chiral symmetry breaking.

It is not completely obvious how the field $\phi_a(x)$ is related to the diquark operator. One way to see it is to couple sources to the diquark field and relate the derivatives of the partition function with respect to the diquark sources to $\phi_a(x)$.

## 3. Sources for the diquark field

The diquark fields defined by equations (2.7) and (2.8) are not gauge invariant. Adding sources for these fields will make sense once a gauge is fixed. To get the effective action in the presence of sources, however, we do not need to fix the gauge. It can be fixed afterwords, when using the effective action to make computations. Let us add to the action (2.2) a source term of the form

$$\sum_x [J_a(x)D^a(x) + J_a^\dagger(x)\bar{D}^a(x)] \,. \tag{3.1}$$

With the same algebraic manipulations of the previous section we find that the only effect of the sources is to replace the functions $\Theta_1(x)$ and $\Theta_2(x)$ entering equation (2.13) by

$$\begin{aligned} \Theta_1^{(J)}(x) &= \Theta_1(x) + 4m(x)[\phi_a^\dagger(x)J_a(x) + J_a^\dagger(x)\phi_a(x) - J_a^\dagger(x)J_a(x)] \,, \\ \Theta_2^{(J)}(x) &= \Theta_2(x) - \frac{1}{9}\Big\{\phi_a^\dagger(x)J_a(x) + J_a^\dagger(x)\phi_a(x) - J_a^\dagger(x)J_a(x) + \\ &\qquad + |\phi(x)|^2[\phi_a^\dagger(x)J_a(x) + J_a^\dagger(x)\phi_a(x)] - \phi_a^\dagger(x)J_a(x)\, J_b^\dagger(x)\phi_b(x)\Big\} \,. \end{aligned}$$



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Taking the derivative respect to $J_a(x)$ and letting the sources vanish, we get the diquark operator expressed in terms of the bosonic fields $\varphi$ and $\phi_a$:

$$D^a(x) = [4\varphi(x)A(x) - \frac{1}{9}(1 - |\phi(x)|^2)B(x)]\,\phi_a^\dagger(x)\,, \tag{3.2}$$

where

$$\begin{aligned} A(x) &= \left(\frac{1}{\tilde{\Theta}_1 - V_B\tilde{\Theta}_2}\right)_{xx}, \\ B(x) &= \left(\frac{1}{\tilde{\Theta}_1 - V_B\tilde{\Theta}_2}V_B\right)_{xx}. \end{aligned} \tag{3.3}$$

In the above expressions matrix multiplication and inversion is understood.

We see, as expected, that the diquark field is proportional to $\phi_a^\dagger$.

## 4. Phase diagram

The effective potential, $\mathcal{U}_{\rm eff}$, to the lowest order semi-classical expansion (mean field approximation) is given by the effective action (2.13) evaluated at constant fields, $\varphi(x) = \bar{\varphi}$ and $|\phi(x)|^2 = v^2$. In this case the determinant entering equation (2.13) can be readily computed by using the Fourier transformation and we have

$$\mathcal{U}_{\rm eff} = v^2 + \frac{3}{4}\bar{\varphi}^2 - \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\ln[\bar{\Theta}_1^2 - \bar{\Theta}_2^2R^2(k)]\,, \tag{4.1}$$

where

$$R^2(k) = \frac{1}{32}\left[-\sum_{\nu=1}^{3}\sin^2\left(\frac{k_\nu}{2}\right) - 2 + e^{6\mu}e^{ik_0} + e^{-6\mu}e^{-ik_0}\right], \tag{4.2}$$

and

$$\begin{aligned} \bar{\Theta}_1 &= \bar{m}(\bar{m}^2 - 4v^2 + 12) \\ \bar{\Theta}_2 &= \frac{1}{36}[4v^4 + v^2\bar{m}^2 + 8v^2 - 3\bar{m}^2 - 12]\,, \end{aligned} \tag{4.3}$$

with $\bar{m} = m_0 + \bar{\varphi}$. A non vanishing expectation value, $v$, of the Higgs field will signal, as usual, the presence of the Higgs phenomenon, which, in this context, implies the diquark condensation.

For the remaining of the paper we will restrict our analysis to the chiral limit, $m_0 = 0$.

Let us analyze the phase structure as a function of the chemical potential. The effective potential (4.1) is invariant under the change $\bar{\varphi} \to -\bar{\varphi}$ (and notice that, by definition, $v \geq 0$). The effective potential (4.1) has the following local minima,

$$\begin{array}{lll} v = 0\,, & \bar{\varphi} = \pm\left(\sqrt{33} - 5\right)^{1/2} & \forall\,\mu \\ v = \sqrt[4]{5}\,, & \bar{\varphi} = 0 & \forall\,\mu \\ v = 0\,, & \bar{\varphi} = 0 & \text{for } \mu > 0.4416 \end{array} \tag{4.4}$$

which are independent of $\mu$.

There is a first order phase transition at $\mu_c \approx 1.557$. For $\mu < \mu_c$ the (degenerate ought to chiral symmetry) absolute minima are at $v = 0$ and $\bar{\varphi} = \pm(\sqrt{33} - 5)^{1/2}$, corresponding to a phase with chiral symmetry spontaneously broken. For $\mu > \mu_c$ the absolute minimum is at $v = 0$ and $\bar{\varphi} = 0$; chiral symmetry is restored in this phase. Obviously, the transition is first order. For $\mu < \mu_c$ the baryon density is zero, and three for $\mu > \mu_c$. Three quarks per lattice site is the maximum allowed by Pauli's principle, so that the transition separates a phase of zero baryon density from a phase saturated of quarks. The system at any intermediate density is thermodynamically unstable and splits into domains of zero and saturated baryon density. This behavior is an artifact of the strong coupling limit and has been observed in other strong coupling analysis at zero temperature and finite chemical potential [28] and [32]–[34].

The minimum at $v = \sqrt[4]{5}$ and $\bar{\varphi} = 0$ is metastable in both phases. It describes a chiral symmetric state with a diquark condensate. The baryon density that correspond to this metastable state is a smooth function of the chiral condensate (see figure 1) that interpolates between zero density and saturation. This means that at any baryon density a state with diquark condensation can be formed and have some short life until the system splits into its zero density and saturated domains. The presence of the metastable state signals the attraction in the quark-quark channel. At strong coupling such attraction is not strong enough to form a stable diquark condensate. The energy difference between this metastable state and the equilibrium state decreases with $\mu$, indicating that the quark-quark attraction becomes the more effective the higher the baryon density (see figure 2). However, the energy difference is nonzero even at high $\mu$ (in the saturated phase).

Had we computed the minimum of the effective potential with diquark sources, we would have obtained as metastable minimum in the limit of vanishing sources

$$\phi_a = v \, \frac{J_a}{|J|} \, . \tag{4.5}$$

Notice that for vanishing sources $J_a/|J|$ merely selects a direction in color space. This does not mean that gauge symmetry is spontaneously broken, since the fluctuations of the redundant gauge variables will destroy the behavior of the classical level. However, if we fix a gauge before including the fluctuations, then the diquark field will have a non vanishing expectation value and the Higgs mechanism will take place.

## 5. Discussion

It is rather natural that diquarks do not condense at strong coupling, since the confining forces are so strong that produce almost pointlike baryons and mesons. The transition driven by the chemical potential separates a phase with zero baryon density from a saturated phase, with three quarks per lattice site. Hence, the system at any finite density is unstable and splits into domains of zero and saturated densities. Deconfinement does not take place at any density and diquark condensation cannot take place in a confined phase.

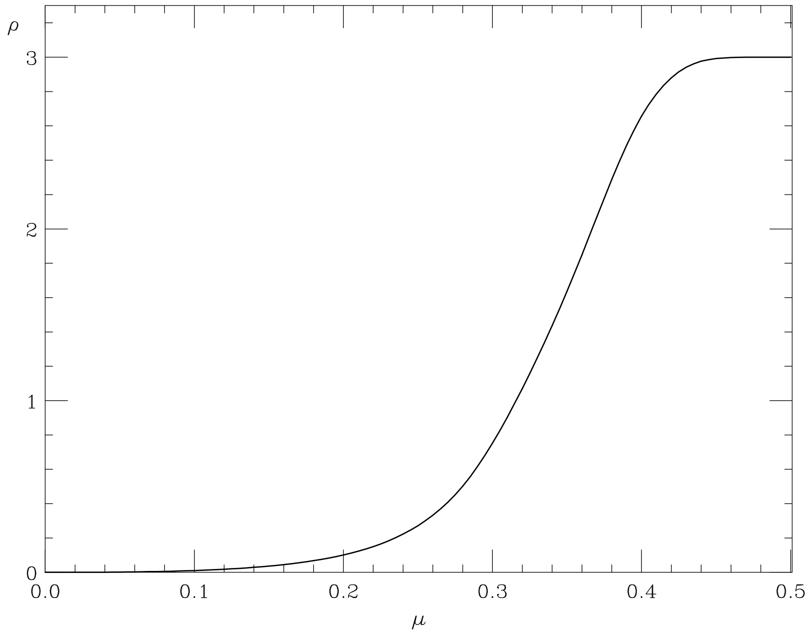


**Figure 1:** Baryon density as a function of the chemical potential in the metastable state with diquark condensation.

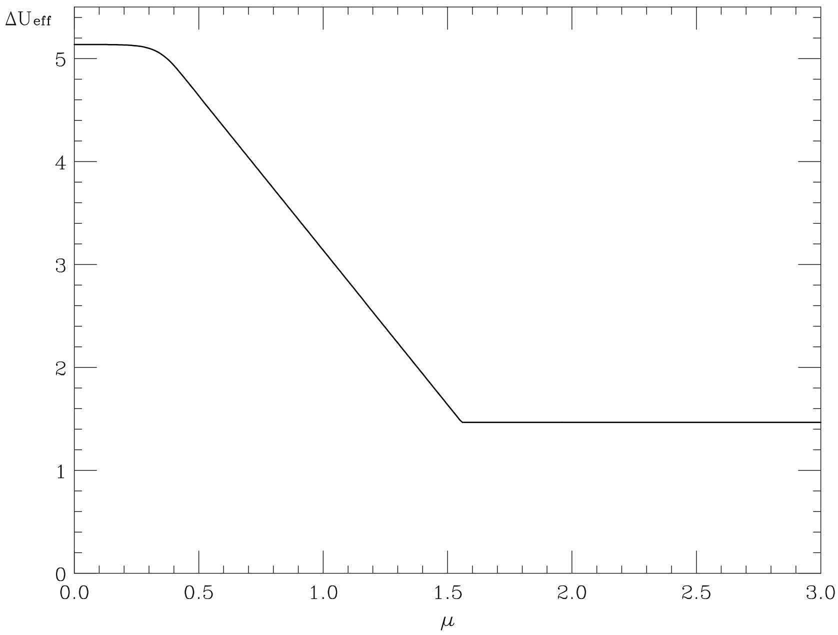


**Figure 2:** Energy difference between the diquark condensate metastable and the equilibrium state.

It is however interesting that a metastable state in which diquark condensation takes place appears at any density. This shows that there is a strong attraction in the quark-quark channel at strong coupling too, where the weak coupling arguments based on one gluon exchange or instanton induced interactions do not apply. The energy difference between the metastable and equilibrium states decreases with the density, and is minimum in the



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saturated regime. The attraction, hence, is the more effective the highest the density. We expect the metastable state to have the usual features induced by diquark condensation: Meissner effect, a gap in the excitation spectrum, and color superconductivity. The crucial question that should be addressed is whether this metastable state becomes stable at sufficiently large density at finite coupling.

A partial answer is the following: assuming that the strong coupling expansion gives meaningful results, diquark condensation cannot take place at sufficiently large couplings, whatever the chemical potential. The reason is that, as we have seen, the energy difference between the state with diquark condensation and the stable state remains positive for any value of the chemical potential in the strong coupling limit. A correction to the effective potential of order $1/g$ cannot remove such energy difference if $g$ is sufficiently large. Hence, diquark condensation cannot take place in the strong coupling region. If, on the other hand, diquarks condense at some finite chemical potential in the weak coupling regime, there must be a critical value of the coupling, $g_c$, such that diquark condensation takes place in some interval of the baryon density for $g < g_c$, but diquarks do not condense at any density for $g > g_c$. This means that the interval of baryon densities (in lattice units) at which diquark condensation occurs as a stable state shrinks as $g$ increases, and vanishes at $g_c$. For $g > g_c$ the state with diquark condensate survives as a metastable state.

It is interesting to note the different behaviour of the chiral transition: since in the strong coupling limit a first order phase transition separates a state with vanishing density and chiral symmetry spontaneously broken from a state saturated of quarks, where chiral symmetry is unbroken, the chiral transition at high density may persist for any value of the coupling, from the weak to the strong coupling regimes. The peculiarity of the strong coupling limit is that the chiral transition occurs at the onset chemical potential (the value of $\mu$ at which the baryon density starts to be non-zero).

For two colors QCD a stable phase characterized by diquark condensation has been found [20]–[25]. Two colors QCD has two peculiarities that make its investigation simpler: 1) the diquark operator is a color singlet and can be used as an order parameter in the usual sense, and 2) the fermion determinant is real and there is no sign problem. The sign problem prevents the applicability of numerical simulations to analyze the phase diagram of three colors QCD and, therefore, strong coupling techniques are useful to improve our understanding of real QCD at finite baryon density.

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